Robust Stochastic Frontier Analysis: a Minimum Density Power Divergence Approach

Federico Belotti[a,*], Giuseppe Ilardi[b]

[a] *CEIS, University of Rome Tor Vergata.*
[b] *Directorate General for Economics, Statistics and Research, Bank of Italy.*

Abstract

In a parametric stochastic frontier framework the statistical model can be represented as

$$y = f(x) \pm u + v$$

where a specific functional form (Cobb-Douglas, *translog*, etc.) is assumed for $f(.)$, the idiosyncratic error v is typically normally distributed and the inefficiency term u is assumed to be generated by some specified parametric one-sided distribution (see Greene, 2008). Models of this kind have become a standard tool in the arsenal of applied economics and are mostly estimated by maximum likelihood methods.

However, it is well known that maximum likelihood estimators are highly sensitive to the presence of extreme values or outliers. To the best of our knowledge, only the nonparametric literature on both deterministic and stochastic frontier models deals with this important issue. Wilson (1993) propose descriptive methods to detect outliers in a nonparametric deterministic frontier framework, Cazals et al. (2002) introduce robust estimators for nonparametric deterministic frontiers and, recently, Florens & Simar (2014) suggest an original and new approach to estimate non parametrically and in a robust way stochastic frontier functions.

In this paper, we propose to exploit the minimum density power divergence framework introduced by Basu et al. (1998) to robustify the estimation of parametric stochastic frontier models for both cross-sectional and panel data. This approach relies on a family of density-based divergences indexed by a non-negative tuning parameter α allowing to control the trade-off between robustness and asymptotic efficiency, thus providing a whole continuum of divergence estimators that begin from the Kullback-Leibler divergence and interpolate to the L_2-distance and beyond. When $\alpha > 0$ these divergences provide a relative-to-the-model downweighting of high leverage observations. Of special note is that this approach can be directly used to estimate a wide range of parametric stochastic frontier specifications, including panel data and heteroskedastic models.

We compare the performance of the minimum density power divergence approach with the maximum likelihood estimator via Monte Carlo simulations in both correctly specified and contaminated stochastic frontier models. Our results show that small values of α produce divergence estimators characterized by strong robustness properties and a negligible loss (relative to maximum likelihood) in terms of efficiency.

Keywords: Stochastic frontier models, Efficiency analysis, Robust estimation, Minimum density power divergence
JEL: C13, C49, D20

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*Corresponding author
Email addresses: federico.belotti@uniroma2.it (Federico Belotti), giuseppe.ilardi@bancaditalia.it (Giuseppe Ilardi)

June 23, 2015

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